CARNEGIE LEARNING MATH SERIES COURSE 1

STUDENT EDITION

VOLUME 1

4TH EDITION

SANDY BARTLE FINOCCHI
WILLIAM S. HADLEY
MARY LOU METZ
MARY LYNN RAITH
JANET SINOPOLI
JACLYN SNYDER

Carnegie Learning
Pittsburgh, Pennsylvania

Carnegie Learning

437 Grant St., Suite 1906
Pittsburgh, PA 15219
Phone 888.851.7094
Customer Service Phone 888.851.7094, option 3

www.carnegielearning.com

Printing History
First Edition 2011
Second Edition 2014
Third Edition 2015
Fourth Edition 2016

ISBN: 978-1-60972-584-6
Set ISBN: 978-1-60972-110-7

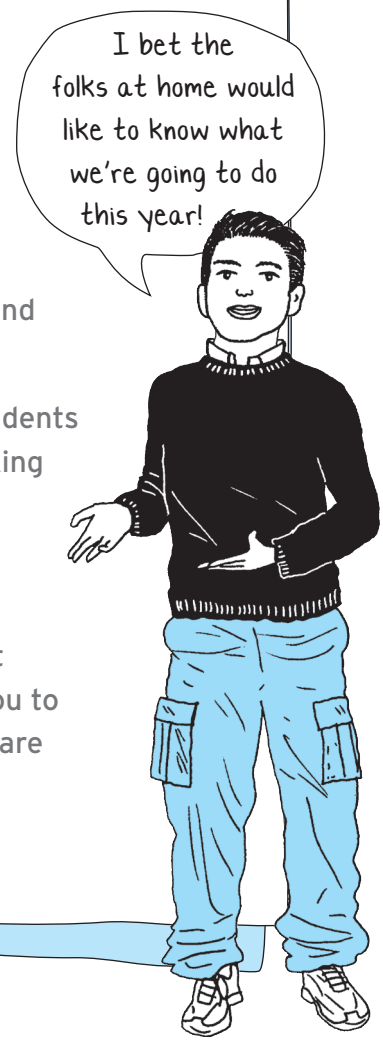
Printed in the United States of America by Cenveo Corporation
1 2 3 4 5 6 7 8 9 CC 18 17 16

Dear Student,

You are about to begin an exciting journey! These mathematical materials were written specifically for *you*, a middle school student. The book you are holding is *your* book. There is lots of space for writing, sketching, drawing, cutting, pasting, and constructing new mathematical ideas. You may want to highlight key terms, take notes in the margins, or even doodle on the cover.

Connections are important in life. The popularity of social networks shows the importance of connections. In much the same way, mathematics connects with so many activities in our lives. Throughout the lessons, you will build new knowledge based upon your prior knowledge. You will apply math to real-world situations so that you can see why it's meaningful. You will encounter models that portray mathematical concepts. Models will be presented in all sorts of ways—from lesson openers, to pictures, to different student methods and approaches to problem solving. You will also use manipulatives, which are objects that you can use to model or reinforce key mathematical concepts.

Of course, if you need additional practice, you can find it in your Assignments and Skills Practice book. Keep in mind, no professional athlete practices by just playing an entire game—ballet dancers repeat some basic steps, moves, and dances; basketball players practice dribbling, shooting, and defending; even writers jot ideas for novels in their spare time—all to improve their skills. Mathematics is no different and these materials enable and encourage you to practice.



Don't worry—you will not be working alone. We encourage students to work together in pairs or in groups because it gets you talking about your insights. Everyone will share his or her ideas and thoughts in class. Sometimes you will help your classmates, and other times they will help you.

Today's workplace demands teamwork and self-confidence. At Carnegie Learning, we have designed a Math Series to help you to make the most of your math course. Enjoy the journey and share your thoughts with others. Have fun while Learning by Doing!

The Carnegie Learning® Curriculum Development Team

ACKNOWLEDGMENTS

Carnegie Learning Curriculum Development Team

- Joshua Fisher
  Math Editor
- David "Augie" Rivera
  Math Editor
- David Dengler
  Director, Curriculum Development
- Jen Gansberger
  Editorial Assistant
- Lezlee Ross
  Curriculum Developer

Advisory Board

- Shelly Allen, Richmond County Schools
- Ryan Baker, Worcester Polytechnic Institute
- Bill Bush, University of Louisville
- John McCook, McCook and Associates
- Roxana Moreno, University of New Mexico
- Doug Rohrer, University of South Florida
- Bob Siegler, Carnegie Mellon University
- Mary Ann Stine, Private Consultant

Vendors

- Bookmasters, Inc.
- Mathematical Expressions
- ESI Design
- Cenveo Corporation

Special Thanks

- Content contributors: Janet Falkowski, Ken Labuskes, Marianne O'Connor, Jennifer Panasko, Agnes Pavolovich
- Peter Arkle for the design and rendering of "The Crew"
- Richmond County School District, Georgia, for piloting lessons and providing implementation feedback
- Carnegie Learning Managers of School Partnership for content and design review
- The Children of Carnegie Learning employees for providing a "middle-schooler's" perspective, with special recognition to:
  - Matthew B.
  - Dawson D.
  - Allison M.
  - Adam, Nic, and Shane R.
  - Aaron and Melinda R.

Photograph Credits

Chapter 1 © istockphoto.com/leezsnow;
Chapter 2 © istockphoto.com/Lingbeek;
Chapter 3 © istockphoto.com/Vasko;
Chapter 4 © istockphoto.com/egdigital;
Chapter 5 © istockphoto.com/LTPHOTOS;
Chapter 6 © istockphoto.com/yenwen;
Chapter 7 © istockphoto.com/Ivan Cholakov;
Chapter 9 © istockphoto.com/Michael Fuller;
Chapter 10 © istockphoto.com/BWBImages;
Chapter 11 © istockphoto.com/narvikk;
Chapter 12 © istockphoto.com/Norman Pogson;
Chapter 13 © istockphoto.com/Diane Diederich;
Chapter 14 © istockphoto.com/Doil82;
Chapter 15 © istockphoto.com/Ccaetano;
Chapter 16 © istockphoto.com/spxChrome

TABLE OF CONTENTS

Table of Contents

3 FRACTIONS 93

Table of Contents

4 DECIMALS 225

Table of Contents

5 RATIOS 305

Table of Contents

8 ALGEBRAIC EXPRESSIONS 529

Table of Contents

THE CREW

The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

MATHEMATICAL REPRESENTATIONS

Introduction

During this course, you will solve problems and work with many different representations of mathematical concepts, ideas, and processes to better understand the world. Each lesson will provide you with opportunities to discuss your ideas, work within groups, and share your solutions and methods with your class. These process icons are placed throughout the text.

Discuss to Understand

- Read the problem carefully.
- What is the context of the problem? Do we understand it?
- What is the question that we are being asked? Does it make sense?
- Is this problem similar to some other problem we know?

Think for Yourself

- Do I need any additional information to answer the question?
- Is this problem similar to some other problem that I know?
- How can I represent the problem using a picture, a diagram, symbols, or some other representation?

Work with Your Partner

- How did you do the problem?
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to each other?
- How will we explain our solution to the class?

Share with the Class

- Here is our solution and the methods we used.
- Are we communicating our strategies clearly?
- We could only get this far with our solution. How can we finish?
- Could we have used a different strategy to solve the problem?

ACADEMIC GLOSSARY

Key Terms of the Course

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas. The Graphic Organizers shown display a definition for a key term, related words, sample questions, and examples.

ANALYZE

DEFINITION

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

RELATED WORDS

- examine
- evaluate
- determine
- observe
- consider
- investigate
- what do you notice?
- what do you think?
- sort and match

ASK YOURSELF

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

EXAMPLES

5. Look at these division problems.

$7\overline{)56}$ $70\overline{)560}$ $700\overline{)5600}$ $7000\overline{)56{,}000}$

a. How are the divisors and dividends in the last three problems related to the first problem?

The divisor and dividends have each been multiplied by 10, 100, and 1000.

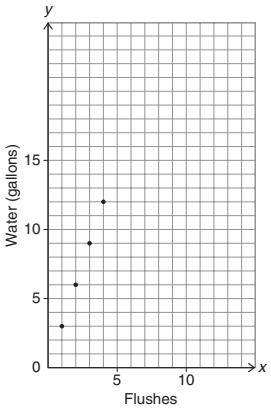
b. Calculate all four quotients. What do you notice about them?

All the quotients are 8. They are all the same.

c. What happens to the quotient when the dividend and divisor are multiplied by the same number?

The quotient remains unchanged.

1. The graph shown represents the number of gallons of water used for the number of times a toilet is flushed.



a. Write each point on the graph as the ratio of *gallons of water used : number of flushes*.

3 gallons of water : 1 flush
6 gallons of water : 2 flushes
9 gallons of water : 3 flushes
12 gallons of water: 4 flushes

b. What do you notice about each ratio?

Each ratio is equivalent, 3 gallons of water : 1 flush.

EXPLAIN YOUR REASONING

DEFINITION

To give details or describe how to determine an answer or solution.
Explaining your reasoning helps justify conclusions.

RELATED WORDS

- show your work
- explain your calculation
- justify
- why or why not?

ASK YOURSELF

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

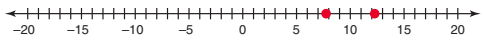
EXAMPLES

10. The Newspaper Club at Marshall Middle School meets every 6 school days. The Math Club meets every 8 school days. Luis is a member of both clubs. He needs to make a plan when both clubs meet on the same school day. Both clubs will meet today after school.

a. After today, when will both clubs meet on the same day again? Explain your reasoning.

Both clubs will both meet 24 school days from today.
$6 = 2 \times 3$
$8 = 2^3$
The LCM is $2^3 \times 3 = 24$.

h. 12.27 (>) 7.75



3. Explain how you knew which rational number was greater in Question 2.

The number to the right was the greater rational number.

REPRESENT

DEFINITION

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

RELATED WORDS

- show
- sketch
- draw
- create
- plot
- graph
- write an equation
- complete the table

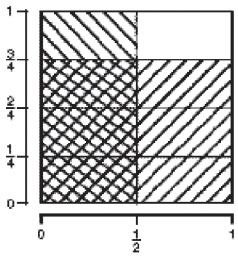
ASK YOURSELF

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

EXAMPLES

2. Represent each product using an area model. Then, state the product.

a. $\frac{3}{4} \times \frac{1}{2}$

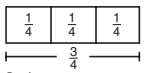


$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

1. Write a sentence to describe what the division expression is asking. Then, draw a diagram to represent the division problem. Finally, calculate the quotient, and write a sentence to describe your answer. Use your fraction strips to help you draw the model.

a. $\frac{3}{4} \div \frac{1}{4}$

How many fourths are in $\frac{3}{4}$?



$\frac{3}{4} \div \frac{1}{4} = 3$

There are three $\frac{1}{4}$ parts in $\frac{3}{4}$.

Academic Glossary

ESTIMATE

DEFINITION

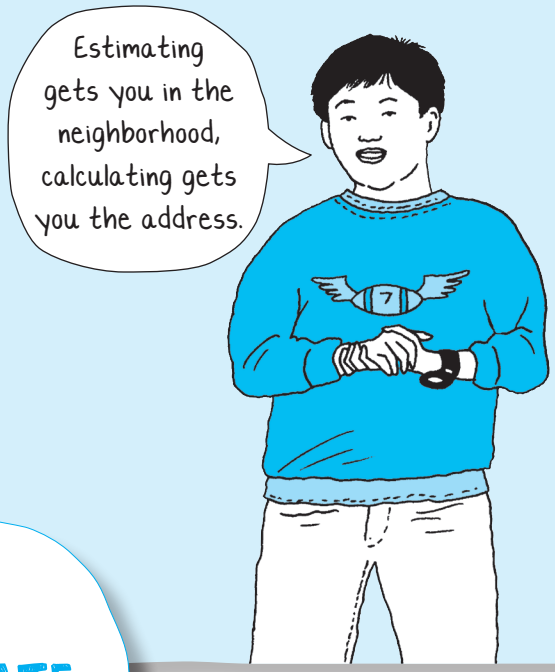
To make an educated guess based on the analysis of given data.
Estimating first helps inform reasoning.

RELATED WORDS

- predict
- approximate
- expect
- about how much?

ASK YOURSELF

- Does my reasoning make sense?
- Is my solution close to my estimation?



EXAMPLES

5. Rewrite each expression using benchmark fractions. Then, estimate the sum. Explain your reasoning.

a. $\frac{8}{9} + \frac{6}{7}$

$1 + 1 = 2$
I know that both fractions are close to 1. So, when I estimate the sum, it is close to, but less than 2.

2. Paul always estimates the total of his purchases at the supermarket. He estimates to ensure he has enough money to pay for his purchases. He also estimates to check that the cashier hasn't made a mistake when ringing up his total.

Today, Paul has these items in his grocery cart.

Bread	$3.25
Peanut Butter	$5.16
Jelly	$2.97
Hot Dogs	$4.86
Hot Dog Buns	$2.42
Mustard	$1.25
Soda	$4.99
Chips	$1.50

a. About how much money does Paul need to pay for his purchases? Explain your reasoning.

Paul needs about $26.
By rounding each item's cost to the nearest dollar, I was able to calculate that
$3 + 5 + 3 + 5 + 2 + 1 + 5 + 2 = 26$.

DEFINITION

To represent or give an account of in words. Describing communicates mathematical ideas to others.

RELATED WORDS

- demonstrate
- label
- display
- compare
- define
- determine
- what are the advantages?
- what are the disadvantages?
- what is similar?
- what is different?

ASK YOURSELF

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

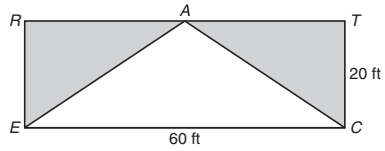
DESCRIBE

EXAMPLES

3. Look at the percents and the decimals you wrote for Question 2 to determine a pattern. Use this pattern to describe how you can write any percent as a decimal.

I can move the decimal point in a percent two places to the left to write the percent as a decimal.

1. The figure shown is composed of a rectangle and triangles.



a. Describe a strategy that can be used to compute the area of the shaded region.

To compute the area of the shaded region, first compute the area of the rectangle. Next, compute the area of triangle *AEC*. Then, subtract the area of triangle *AEC* from the area of rectangle *RECT*.

Academic Glossary

Problem Types You Will See

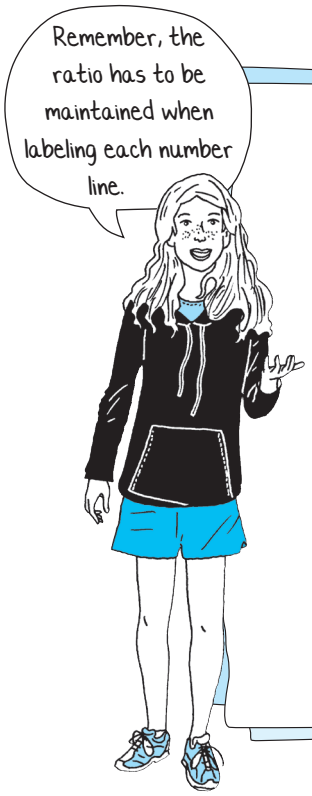
Worked Example

When you see a Worked Example:

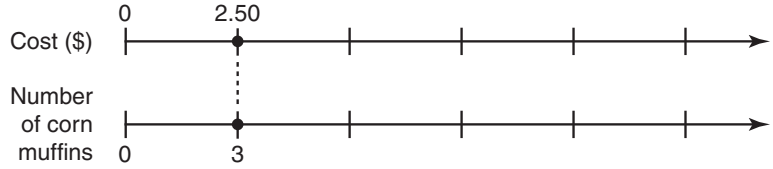
- Take your time to read through it,
- Question your own understanding, and
- Think about the connections between steps.

Ask yourself:

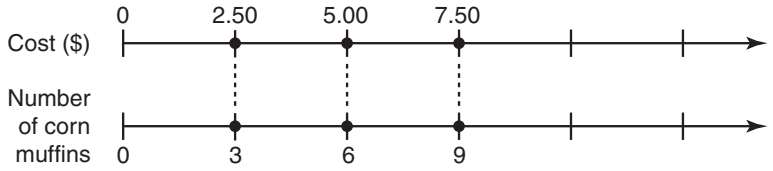
- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?



The ratio $2.50 : 3 corn muffins is shown on the double number line.



You can see other equivalent ratios of *cost : number of corn muffins* by continuing to label each interval.



1. State the two new ratios of cost : number of corn muffins shown on the second double number line.

   $5.00 : 6 corn muffins made

   $7.50 : 9 corn muffins made

2. Describe the interval represented on each number line.

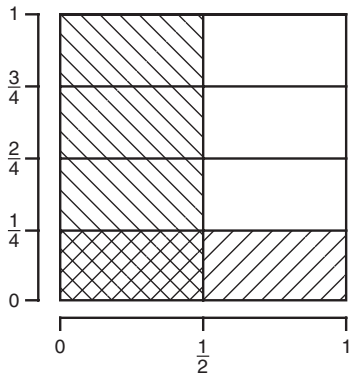
   The interval for the cost number line is $2.50.

   The interval for the number of corn muffins made number line is 3.

Problem Types

Let's consider an area model for $\frac{1}{4} \times \frac{1}{2}$ and what it represents.

To represent $\frac{1}{4}$ along one side of the square, divide the square into four equal parts along the vertical line. Then shade $\frac{1}{4}$.



To represent $\frac{1}{2}$ along the other side, divide the square along the horizontal line into two equal parts. Then, shade $\frac{1}{2}$.

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

The area of the overlapping region is the product of the fractions.

Problem Types

Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the *correct* solution.
- Think about the connections between steps.

Ask yourself:

- Why is this method correct?
- Have I used this method before?

Problem Types

Kaye

I used the weights for a 30-lb person and a 90-lb person to obtain the weight of a 120-lb person.

Weight on Earth (lbs)	60	30	90	120
Weight on the moon (lbs)	10	5	15	20

So that means 120 lbs on Earth : 20 lbs on the moon.

a. Explain Kaye's reasoning.

Kaye knew that 30 lbs on Earth plus 90 lbs on Earth gave her 120 lbs on Earth. So, she also added the corresponding weights on the moon—5 lbs plus 15 lbs to get 20 lbs.

Thumbs Down

When you see a Thumbs Down icon:

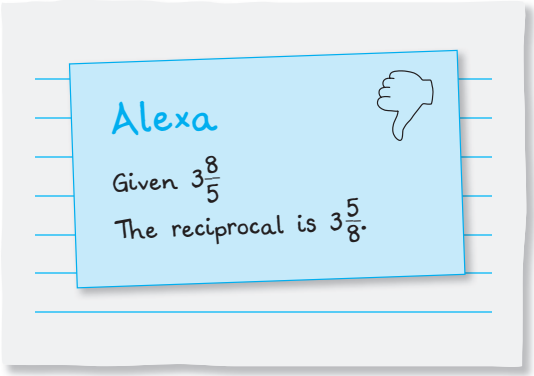
- Take your time to read through the *incorrect* solution.
- Think about what error was made.

Ask yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

8. Alexa wrote the reciprocal of the mixed number incorrectly. Explain why she is incorrect and provide the correct reciprocal.

Alexa did not take the reciprocal of the entire mixed number. She must first convert $3\frac{5}{8}$ to the improper fraction $\frac{29}{8}$, and then take the reciprocal, which is $\frac{8}{29}$.



Problem Types

Who's Correct?

When you see a Who's Correct? icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine correct or not correct.

Ask yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

4. Kaye said, "I see another equivalent ratio when I look at the way Carla showed her work."

30 lbs on Earth : 5 lbs on the moon
120 lbs on Earth : 20 lbs on the moon
150 lbs on Earth : 25 lbs on the moon

Is Kaye correct? Explain her reasoning.

Kaye is correct. She added the corresponding parts of each equivalent ratio.

Problem Types